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XXII. ΚΟΣΚΙΝΟΝ ΕΡΑΤΟΣΘΕΝΟΥΣ.

OR,

The Sieve of Eratosthenes.

Being an account of his method of finding all the Prime Numbers, by the Rev. Samuel Horsley, *F. R. S.*

Read May 7, 1772.

A Prime number is such a one, as hath no intregral divisor but unity.

A number, which hath any other integral divisor, is Composite.

Two or more numbers, which have no common integral divisor, besides unity, are said to be Prime with respect to one another.

Two or more numbers, which have any common integral divisor besides unity, are said to be Composite with respect to one another.

The distinction of numbers into Prime and Composite, is so generally understood, that I suppose it is needless to enlarge upon it.

To determine, whether several numbers proposed be Prime or Composite *with respect to one another*, is an easy Problem. The solution of it is given by Euclid, in the three first propositions of the 7th book

book of the Elements, and is to be found in many common treatises of Arithmetic and Algebra. But to determine, concerning any number proposed, whether it be *absolutely* Prime or Composite, is a Problem of much greater difficulty. It seems indeed incapable of a direct solution, by any general method; because the successive formation of the prime numbers doth not seem reducible to any general law. And for the same reason, no direct method hath hitherto been hit upon, for constructing a Table of all the prime numbers to any given limit. Eratosthenes, whose skill in every branch of the philosophy and literature of his times, rendered his name so famous among the Sages of the Alexandrian School, was the inventor of an indirect method, by which such a table might be constructed, and carried to a great length, in a short time, and with little labour. This extraordinary and useful invention is at present, I believe, little, if at all, known; being described only by two writers, who are seldom read, and by them but obscurely; by Nicomachus Gerasinus, a shallow writer of the 3d or 4th century, who seems to have been led into mathematical speculations, not so much by any genius for them, as by a fondness for the mysteries of the Pythagorean and Platonic philosophy; and by Boethius, whose treatise upon numbers is but an abridgment of the wretched performance of Nichomachus *. I flatter myself therefore, that a succinct account of it will not be unacceptable to this learned Society.

* There are more pieces than one of this Nichomachus extant. That which I refer to is intitled Εισαγωγη Αριθμητικη.

But before I enter exprefsly upon the fubject, I muft take the liberty to animadvert upon a certain Table, which, among other pieces afcribed to Eratofthenes, is printed at the end of the beautiful edition of Aratus publifhed at Oxford in the year 1672, and is adorned with the title of Κοσκινον Ερατοσθενους. It contains all the odd numbers from 3 to 113 inclufive, diftributed in little cells, all the divifors of every Compofite number being placed over it, in its proper cell, and the Prime numbers are diftinguifhed, fo far as the table goes, by having no divifors placed over them. It hath probably been copied either from a Greek comment upon the Arithmetic of Nicomachus, preferved among the manufcripts of Mr. Selden in the Bodleian Library, in which, though the manufcript is now fo much decayed as to be in moft places illegible, I find plain veftiges of fuch a table *, which might be more perfect 100 years ago, when the Oxford Aratus was publifhed; or elfe, from another comment, tranflated from a Greek manufcript into Latin, and publifhed in that language, by Camerarius, in which a table of the very fame form occurs, extending from the number 3 to 109 inclufive. It may fufficiently fkreen the editor of Aratus from cenfure, that he had thefe authorities to publifh this table as the Sieve of Eratofthenes; efpecially as they are in fome meafure fupported by paffages of Nicomachus himfelf. But the Sieve of Eratofthenes was quite another thing.

* This manufcript feems to have contained the text of Nicomachus with Scholia in the margin. But the table evidently belongs to the Scholia, not to the text.

The Oxford editor hath annexed to his table, to explain the uſe of it, ſome detached paſſages, which he hath ſelected from the text of Nicomachus, and from a comment upon Nicomachus aſcribed to Joannes Grammaticus. In theſe paſſages the difference between Prime and Compoſite numbers is explained, in many words indeed, but not with the greateſt accuracy; and it is propoſed to frame a kind of Table of all the odd numbers, from 3 to any given limit, in which the Compoſite numbers ſhould be diſtinguiſhed by certain marks *. The Primes would conſequently be characteriſed, as far as the table ſhould be carried, by being unmarked. But, upon what principles, or by what rule, ſuch a table is to be conſtructed, is not at all explained. It is obvious that, in order to *mark* the Compoſite numbers, it is neceſſary to know which are ſuch. And, without ſome rule to diſtinguiſh which numbers are Prime, and which are Compoſite, independent of any table in which they ſhall be diſtinguiſhed by marks, it is impoſſible to judge, whether the table be true, as far as it goes, or to extend it, if requiſite, to a further limit. Now it was the Rule by which the Prime numbers and the Compoſite might be diſtinguiſhed, not a Table conſtructed we know not how, that was the invention of Eratoſthenes, to which from its uſe, as well as from the nature of the operation, which

* Nicomachus and Joannes Grammaticus propoſe that theſe marks ſhould be ſuch, as ſhould not only diſtinguiſh the compoſite numbers, but likewiſe ſerve to expreſs all the diviſors of every ſuch number. It will be ſhewn, in a proper place, that this was no part of the original contrivance of the Sieve.

proceeds (as will be ſhewn) by a gradual extermination of the compoſite numbers from the arithmetical ſeries 3. 5. 7. 9. 11. &c. infinitely continued, its author gave the name of the Sieve. I have thought it neceſſary to premiſe theſe remarks, to remove a prejudice, which I apprehend many may have conceived, as this beautiful and valuable edition of Aratus is in every ones hands, that this ill-contrived table, the uſeleſs work of ſome monk in a barbarous age, was the whole of the invention of the great Eratoſthenes, and in juſtice to myſelf, that I might not be ſuſpected of attempting to reap another's harveſt.

I now proceed, to give a true account of this excellent invention; which, for its uſefulneſs, as well as for its ſimplicity, I cannot but conſider as one of the moſt precious remnants of Ancient Arithmetic. I ſhall venture to repreſent it according to my own ideas, not obliging myſelf to conform, in every particular, to the account of Nicomachus, which I am perſuaded is in many circumſtances erroneous. In ſtating the principles upon which the Operation of the Sieve was founded, he hath added obſervations upon certain relations of the odd numbers to one another, which are certainly his own, becauſe they are of no importance in themſelves, and are quite foreign to the purpoſe. Every thing of this kind I omit: and having ſtated what I take to have been the genuine Theory of Eratoſthenes's method, cleared from the adulterations of Nicomachus, I deduce from it an operation of great ſimplicity, which ſolves the Problem in queſtion with wonderful eaſe, and which,

 becauſe

becaufe it is the moft fimple that the theory feems to afford, I fcruple not to adopt as the original Operation of the Sieve, though nothing like it is to be found in Nicomachus; though, on the contrary, Nichomachus, and all his Commentators, would fuggeft an operation very different from it, and far more laborious. For the fatisfaction of the curious and the learned, I have annexed a copy of fo much of Nicomachus's treatife, as relates to this fubject, with fuch corrections of the text, as it ftands in the edition of Wichelius, printed at Paris ann. 1538, as the fenfe hath fuggefted to me, or I have thought proper to adopt, upon the authority of a manufcript preferved among thofe of Archbifhop Laud, in the Bodleian Library; which, in this part, I have carefully collated. By comparing this with the account which I fubjoin, every one will be able to judge how far I have done juftice to the invention I have undertaken to explain.

PROBLEM.

To find all the Prime Numbers.

The number 2 is a Prime number; but, except 2, no even number is Prime, becaufe every even number, except 2, is divifible by 2, and is therefore Compofite. Hence it follows, that all the Prime numbers, except the number 2, are included in the feries of the odd numbers, in their natural order, infinitely extended; that is, in the feries

3. 5. 7. 9. 11. 13. 15. 17. 19. 21. 23. 25. 27. 29. 31. 33. 35. 37. 39. 41. 43. 45. 47. 49. 51. &c.

Every

Every number which is not Prime, is a multiple of ſome Prime number, as Euclid hath demonſtrated (Element. 7. prop. 33.) Therefore the foregoing ſeries conſiſts of the Prime numbers, and of multiples of the Primes. And the multiples, of every number in the ſeries, follow at regular diſtances; by attending to which circumſtance, all the multiples, that is, all the Compoſite numbers, may be eaſily diſtinguiſhed and exterminated.

I ſay, the multiples of all numbers, in the foregoing ſeries, follow at regular diſtances.

For between 3 and its firſt multiple in the ſeries (9) two numbers intervene, which are not multiples of 3. Between 9 and the next multiple of 3 (15) two numbers likewiſe intervene, which are not multiples of 3. Again between 15 and the next multiple of 3 (21) two numbers intervene, which are not multiples of 3; and ſo on. Again, between 5 and its firſt multiple (15) four numbers intervene, which are not multiples of 5. And between 15 and the next multiple of 5 (25) four numbers intervene which are not multiples of 5; and ſo on. In like manner, between every pair of the multiples of 7, as they ſtand in their natural order in the ſeries, 6 numbers intervene which, are not multiples of 7. Univerſally, between every two multiples of any number n, as they ſtand in their natural order in the ſeries, $\overline{n-1}$ numbers intervene, which are not multiples of n.

Hence may be derived an Operation for exterminating the Compoſite numbers, which I take to have been the Operation of the Sieve, and is as follows.

The Operation of the Sieve.

Count all the terms of the ſeries following the number 3, by threes, and expunge every third number. Thus all the multiples of 3 are expunged. The firſt uncancelled number that appears in the ſeries, after 3, is 5. Expunge the ſquare of 5. Count all the terms of the ſeries, which follow the ſquare of 5, by fives, and expunge every fifth number, if not expunged before. Thus all the multiples of five are expunged, which were not at firſt expunged, among the multiples of 3. The next uncancelled number to 5 is 7. Expunge the ſquare of 7. Count all the terms of the ſeries following the ſquare of 7, by ſevens, and expunge every ſeventh number, if not expunged before. Thus all the multiples of 7 are expunged, which were not before expunged among the multiples of 3 or 5. The next uncancelled number which is now to be found in the ſeries, after 7, is 11. Expunge the ſquare of 11. Count all the terms of the ſeries, which follow the ſquare

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3. 5. 7. ~~9~~. 11. 13. ~~15~~. 17. 19. ~~21~~. 23. ~~25~~. ~~27~~. 29. 31.
~~33~~. ~~35~~. 37. ~~39~~. 41. 43. ~~45~~. 47. ~~49~~. ~~51~~. 53. ~~55~~. ~~57~~. 59.
61. ~~63~~ ~~65~~. 67. ~~69~~. 71. 73. ~~75~~. ~~77~~. 79. ~~81~~. 83. ~~85~~. ~~87~~.
89. ~~91~~. ~~93~~. ~~95~~. 97. ~~99~~. 101. 103. ~~105~~. 107. 109. ~~111~~. 113.
~~115~~. ~~117~~. ~~119~~. ~~121~~. ~~123~~. ~~125~~. 127. ~~129~~. 131. ~~133~~. ~~135~~.
137. 139. ~~141~~. ~~143~~. ~~145~~. ~~147~~. 149. 151. ~~153~~. ~~155~~. 157.

of 11, by elevens, and expunge every eleventh number, if not expunged before. Thus all the multiples of 11 are expunged, which were not before expunged among the multiples of 3, 5, and 7. Continue these expunctions, till the first uncancelled number that appears, next to that whose multiples have been last expunged, is such, that its square is greater than the last and greatest number to which the series is extended. The numbers which then remain uncancelled are all the Prime numbers, except the number 2, which occur in the natural progression of number from 1 to the limit of the series. By the limit of the series I mean the last and greatest number to which it is thought proper to extend it.

Thus the prime numbers are found to any given limit.

Nicomachus proposes to make such marks over the Composite numbers, as should shew all the divisors of each. From this circumstance, and from the repeated intimations both of Nicomachus, and his commentator Joannes Grammaticus *, one would be led to imagine, that the Sieve of Eratosthenes was something more than its name imports, a method of sifting out the Prime numbers from the indiscriminate mass of all numbers Prime and Composite, and that, in some way or other, it exhibited all the divisors of every Composite number, and likewise shewed whether two or

* The Comment of Joannes Grammaticus is extant in manuscript in the Savilian Library at Oxford, to which I have frequent access, by the favour of the Reverend and Learned Mr. Hornsby, the Savilian Posessor of Astronomy.

more

more Compoſite numbers were Prime or Compoſite with reſpect to each other. I have many reaſons to think, that this was not the caſe. I ſhall as briefly as poſſible point out ſome of the chief, for the matter is not ſo important, as to juſtify my troubling the Society with a minute detail of them. Firſt then, in the natural ſeries of odd numbers, 3. 5. 7. &c. every number is a diviſor of ſome ſucceeding number. Therefore if we are to have marks for all the different diviſors of every Compoſite number, we muſt have a different mark for every odd number. Therefore we muſt have as many marks, or ſyſtems of marks, as numbers; and I do not ſee, that it would be poſſible, to find any more compendious marks, than the common numeral characters. This being the caſe, it would be impracticable to carry ſuch a table as Nicomachus propoſes, and his commentators have ſketched, to a ſufficient length to be of uſe, on account of the multiplicity of the diviſors of many numbers, and the confuſion which this circumſtance would create *. It is hardly to be ſuppoſed, that Eratoſthenes could overlook this obvious difficulty, though Nicomachus hath not attended to it. Eratoſthenes therefore could not intend the conſtruction of ſuch a table.

In the next place, ſuch a table not being had, Eratoſthenes could not but perceive, that, the determining whether two or more numbers be Prime or Compoſite with reſpect to one another, is in all caſes to be done more eaſily, by the direct method given by Euclid, than by

* The number 3465 hath no leſs than 22 different diviſors.

the method of the Sieve. And he could not mean, to apply this method to a problem, to which another was better adapted.

Laſtly, Eratoſthenes could not mean, that the method of the Sieve ſhould be applied to the finding of all the poſſible diviſors of any Compoſite number propoſed, becauſe he could not be unacquainted with a more ready way of doing this, founded upon two obvious Theorems, which could not be unknown to him.

The Theorems I mean are theſe.

1ſt. If two Prime numbers multiply each other, the number produced hath no diviſors but the two prime factors.

2d. If a Prime number multiply a Compoſite number, and likewiſe multiply all the diviſors of that compoſite ſeverally, the numbers produced by the multiplications of theſe diviſors will be diviſors of the number produced by the firſt multiplication: And the number produced by the firſt multiplication will have no diviſors, but the two factors, the diviſors of the Compoſite factor, and the numbers made by the multiplication of theſe diviſors by the Prime factor ſeverally.

The method of finding all the diviſors of any Compoſite number, delivered by Sir Iſaac Newton in the Arithmetica Univerſalis, and by Mr. Maclaurin in his Treatiſe of Algebra, may be deduced from theſe propoſitions, as every mathematician will eaſily perceive. This method requires indeed that the leaſt prime diviſor ſhould be previouſly found; and, if the leaſt prime diviſor ſhould happen to be a large number, as it is not aſſignable by any general method, the investigation of it by repeated tentations may be very tedious. A table therefore of the odd numbers *, in which the Composite numbers should each have its least Prime divisor written over it, would be very useful. But Nichomachus's project of framing a table in which each Composite number should have *all* its divisors written over it, is ridiculous and absurd, on account of the insuperable difficulties which would attend the execution of it.

Feb. 7, 1772.

S. Horsley.

* A table of the odd numbers would be sufficient: for the number 2 is the least prime divisor of every even number; and it is easy, even in the largest numbers, to try whether they are divisible by 2. In our method of notation, this may always be known, by observing the last figure in the expression of the number proposed.

EXCERPTA QUÆDAM
EX
Arithmeticâ Nicomachi
Ad Cribrum Eratosthenis pertinentia.

Ἡ δὲ τούτων γένεσις (*a*), ὑπὸ Ἐρατοσθένους, καλεῖται Κόσκινον· ἐπειδὴ ἀναπεφυρμένους τοὺς περισσοὺς λαβόντες καὶ ἀδιακρίτους, ἐξ αὐτῶν [τὰ διαφέροντα ἀλλήλων εἴδη](*b*) ταύτῃ τῇ τῆς γενέσεως (*c*) μεθόδῳ διαχωρίζομεν, ὡς δι' ὀργάνου ἢ κοσκίνου τινός· καὶ ἰδίᾳ μὲν τοὺς πρώτους καὶ ἀσυνθέτους, χωρὶς δὲ τοὺς μίκτους εὑρίσκομεν. Ἔστι δὲ ὁ τρόπος τοῦ Κοσκίνου τοιοῦτος. Ἐκθέμενος τοὺς ἀπὸ τριάδος πάντας ἐφεξῆς περισσοὺς, ὡς δυνατὸν μάλιστα ἐπὶ μήκιστον στίχον, ἀρξάμενος ἀπὸ τοῦ πρώτου, ἐπισκοπῶ τίνας οἷός τε ἔσται μετρεῖν ἕκαστος· καὶ εὑρίσκω δυνατὸν ὄντα τὸν πρῶτον, ἤτοι τὸν γ̄, τοὺς δύο μέσους διαλείποντας (*d*) μετρεῖν, μέχρις οὗ προχωρεῖν ἐθέλωμεν (*e*). οὐχ ὡς ἔτυχε δὲ, καὶ εἰκῇ, μετροῦντα, ἀλλὰ τὸν μὲν πρώτως αὐτῶν κείμενον, τοῦτ' ἔστι τὸν ἀφ' ἑαυτοῦ τοὺς δύο μέσους διαλεί-

(*a*) Mallem εὕρεσις, etsi, ne quid dissimulem, lectioni receptæ adstipulatur Boethii interpretatio.

(*b*) Voces uncis inclusas conjecturâ supplevi; quin et sequentium ordinem paululum immutavi, pro τῇ γενέσεως μεθόδῳ ταύτῃ, scribendo ταύτῃ τῇ κ. τ. λ.

(*c*) Vocem γενέσεως hic loci retinendam censeo. Locum integrum sic interpretor. " Suam horum indaginem Eratosthenes, Cribrum vocavit. Propterea quod imparibus universis, nullo generum discrimine, in medio collocatis, ipsam procreationem continuam, quo tradidit ille modo, insequendo [id est, procreationis continuæ, Eratosthenis modo, exploratâ lege] species diversas seorsim sistimus, cribro tanquam separatas."

(*d*) Cod. MS. habet διαλείποντα. Wechelius παραλείποντα.

(*e*) Ex Cod. MS. pro ἐθέλομεν.

ποντα (*f*), κατὰ τὴν τοῦ πρωτίστου ἐν τῷ στίχῳ κειμένου ποσότητα μετρήσει· τοῦτ᾽ ἔστι κατὰ τὴν ἑαυτοῦ, τρὶς γάρ· τὸν δ᾽ ἀπ᾽ ἐκείνου δύο διαλείποντα, κατὰ τὴν τοῦ δευτέρου τεταγμένου, πεντάκις γάρ· τὸν δὲ περαιτέρω πάλιν δύο διαλείποντα, κατὰ τὴν τοῦ τρίτου τεταγμένου, ἑπτάκις γάρ· τὸν δὲ ἔτι περαιτέρω ὑπὲρ δύο κείμενον, κατὰ τὴν τοῦ τετάρτου τεταγμένου, ἐννεάκις γάρ· καὶ ἐπ᾽ ἄπειρον τῷ αὐτῷ τρόπῳ. Εἶτα μετὰ τοῦτον, ἀπ᾽ ἄλλης ἀρχῆς, ἐπὶ τὸν δεύτερον ἐλθών, σκοπῶ τίνας οἷός τε ἔστι μετρεῖν· καὶ εὑρίσκω πάντας τοὺς τέσσαρας (*g*) διαλείποντας· ἀλλὰ τὸν μὲν πρῶτον, κατὰ τὴν ἐν τῷ στίχῳ πρωτίστου τεταγμένου ποσότητα· τρὶς γάρ. τὸν δὲ δεύτερον, κατὰ τὴν τοῦ δευτέρου· πεντάκις γάρ· τὸν δὲ τρίτον, κατὰ τὴν τοῦ τρίτου· ἑπτάκις γάρ· καὶ τοῦτο ἐφεξῆς ἀεί. Πάλιν δὲ ἄνωθεν, ὁ τρίτος, ὁ ζ῀, τὸ μετρεῖν * παραλαβών, μετρήσει τοὺς ἓξ διαλείποντας· ἀλλὰ τὸν μὲν πρώτιστον, κατὰ τὴν τοῦ γ῀ (*h*) ποσότητα, πρώτου κειμένου· τὸν δὲ δεύτερον κατὰ τὴν τοῦ ε῀· δευτεροταγὴς γὰρ οὗτος (*i*). τὸν δὲ τρίτον, κατὰ τὴν τοῦ ζ῀, τρίτην γὰρ ἔχει (*k*) οὗτος τάξιν ἐν τῷ στίχῳ. καί, κατὰ τὴν αὐτὴν ἀναλογίαν, δι᾽ ὅλου (*l*) ἀπαραποδίστως (*m*) προχωρήσει σοι τοῦτο, ὥστε τὸ μὲν μετρεῖν διαδέξονται, κατὰ τὴν ἐν τῷ στίχῳ αὐτῶν ἐγκειμένην τάξιν· τὸ δὲ πόσους διαλείποντας,

(*f*) Locum in Editione Wechelii corruptum, in Cod. MS. mutilum & turbatum, conjecturâ, prout potui, sanatum dedi. Editio Wechelii habet τὸν τοὺς δύο μέσους ὑπερβαίνοντα. Codex MS. τὸν δύο. τελέσει τὸν τρία.

(*g*) Conjecturâ, pro τετράδι.

(*h*) Litera numeralem γ῀, conjecturâ posui pro voce τρια.

(*i*) Restitui ex Cod. MS pro ὄντος, quæ est Wechelii lectio.

(*k*) Particulam καὶ omisi.

(*l*) Wechelium sequor. Cod. MS. habet λογου, sensu, ut videtur, nullo.

(*m*) Ex Cod. MS. pro ἀπαρεμπόδιστον.

* Conjecturâ pro μετρον.

κατὰ τὴν ἀπὸ δυάδος ἐπ' ἄπειρον εὔτακτον τῶν (n) ἀρτίων προκοπὴν, ἢ κατὰ τὴν τοῦ χώρας διπλασίασιν καθ' ἣν ὁ μετρῶν τέτακται· τὸ δὲ ποσάκις, κατὰ τὴν τῶν ἀπὸ τριάδος περισσῶν εὔτακτον ἐπ' ἄπειρον (o) προχώρησιν (p). Ἐὰν οὖν σημείοις τισὶν ἐπιστίζῃς τοὺς ἀριθμοὺς, εὑρήσεις τοὺς μεταλαμβάνοντας τὸ μετρεῖν, οὔτε ἅμα πάντας τὸν αὐτόν ποτε μετροῦντας, ἔστι δὲ ὅτε οὐδὲ δύο τὸν αὐτόν· οὔτε πάντας ἁπλῶς τοὺς ἐκκειμένους ὑποπίπτοντας μέτρῳ τινὶ αὐτῶν. ἀλλὰ τινὰς μὲν παντελῶς διαφεύγοντας τὸ μετρηθῆναι ὑφ' οὑτινοσοῦν· τινὰς δὲ ὑφ' ἑνὸς μόνου μετρουμένους· τινὰς δὲ ὑπὸ δυὸ, ἢ καὶ πλειόνων. Οἱ μὲν οὖν μηδαμῶς (q) μετρηθέντες, ἀλλὰ διαφυγόντες τοῦτο, πρῶτοι εἰσὶ καὶ ἀσύνθετοι, ὡς ὑπὸ κοσκίνου διακριθέντες. οἱ δὲ ὑφ' ἑνὸς μόνου μετρηθέντες, κατὰ τὴν ἑαυτοῦ (r) ποσότητα, ἓν μόνον μόριον ἑτερώνυμον ἕξουσι πρὸς τῷ παρωνύμῳ. οἱ δὲ ὑφ' ἑνὸς μὲν (s), ἑτέρου δὲ ποσότητι, καὶ μὴ τῇ ἑαυτοῦ, ἢ ὑπὸ δύο ὁμοῦ μετρηθέντες, πλείονα ἕξουσι τὰ ἑτερώνυμα μέρη πρὸς τῷ παρωνύμῳ. τοῦτοι οὖν ἔσονται

(n) Conjectura pro τὴν.

(o) Voces ἐπ' ἄπειρον ex Cod. MS. restitui.

(p) Nempe series numerorum imparium 3, 5, 7, 9, &c. infinite protensa, cum numeros impares universos contineat, imparis cujusvis multiplices omnes impares necessario complectitur. Esto igitur n numerus quilibet impar. In serie 3, 5, 7, &c. infinite protensâ, habes numeros omnes $\overline{n\times 3}$, $\overline{n\times 5}$, $\overline{n\times 7}$, $\overline{n\times 9}$, &c. Et cum seriei ea Lex sit & Conditio, ut naturali ordine numeri impares sequantur, & minor omnis numerus majorem præcedat, fieri nequit, quin multiplices numeri n eum inter se ordinem servent, ut minor quisque majorem præcedat. Primus igitur erit $\overline{n\times 3}$, secundus $\overline{n\times 5}$, tertius $\overline{n\times 7}$, & universim, $\overline{n\times m}$ eum habiturus est, inter multiplices, locum, quem numerus m in serie.

(q) Ex Cod. MS. vice οὐδαμῶς, quæ Wechelii lectio est.

(r) Conjecturâ pro ἑαυτῶν.

(s) Particulam μὲν ex Cod. MS restitui.

δεύτεροι καὶ σύνθετοι. Τὸ δὲ τρίτον μέρος, τὸ κοινὸν ἀμφοτέρων, ὁ καθ' ἑαυτὸ μὲν δεύτερον καὶ σύνθετον, πρὸς ἄλλο δὲ πρῶτον καὶ ἀσύνθετον, ἔσονται) ἀποτελούμενοι ἀριθμοὶ, κατὰ τὴν ἑαυτοῦ ποσότητα πρώτου καὶ ἀσυνθέτου μετρήσαντος τινὸς, εἴτις [τούτῳ τῷ τρόπῳ] (*t*) γινόμενος, συγκρίνοιτο πρὸς ἄλλον ὡσαύτως τὴν θέσιν ἔχοντα. ὥσπερ ὁ θ̄, ἐγένετο γὰρ ἐκ τοῦ γ̄ (*u*) κατὰ τὴν ἑαυτοῦ ποσότητα μετρήσαντος· τρὶς γάρ· εἰ συγκρίνοιτο πρὸς τὸν κε̄· ἐγένετο γὰρ καὶ οὗτος (*x*) ἐκ τοῦ ε̄, κατὰ τὴν ἑαυτοῦ ποσότητα μετρήσαντος· πεντάκις γάρ· κοινὸν μέτρον τούτοις οὐκ ἔσται, εἰ μὴ μόνη ἡ Μονάς.

(*t*) Voces τούτῳ τῷ τρόπῳ conjecturâ supplevi.

(*u*) Literam numeralem γ̄ pro voce τρίτου quæ apud Wechelium legitur, ex Cod. MS restitui.

(*x*) Voces γὰρ καὶ οὗτος ex Cod. MS. restitui.

Ex

Ex Arithmeticâ Boethii.

Lib. I. c. xvii.

GENERATIO autem ipſorum atque ortus hujuſmodi inveſtigatione colligitur, quam ſcilicet Eratoſthenes Cribrum nominabat; quod cunctis imparibus in medio collocatis, per eam, quam tradituri ſumus, artem, qui primi, quive ſecundi, quique tertii generis videantur eſſe diſtinguitur. Diſponantur enim a ternario numero cuncti in ordinem impares, in quamlibet longiſſimam porrectionem 3. 5. 7. 9. 11. 13. 15. 17. 19. 21. 23. 25. 27. 29. 31. 33. 35. 37. 39. 41. 43. 45. 47. 49. His igitur ita diſpoſitis, conſiderandum, primus numerus quem eorum, qui ſunt in ordine poſiti, primum metiri poſſit: ſed, duobus prætereritis, illum, qui poſt eos eſt poſitus, mox metitur: et, ſi poſt eundem ipſum quem menſus eſt, alii duo tranſmiſſi ſunt, illum, qui poſt duos eſt, rurſus metitur: et, eodem modo ſi duos quis reliquerit, poſt eos qui eſt, a primo numero metiendus eſt; eodemque modo, relictis ſemper duobus, a primo, in infinitum pergentes metientur. Sed id non vulgo neque confuſe. Nam primus numerus illum, qui eſt poſt duos ſecundum ſe locatos, per ſuam quantitatem metitur: ternarius enim numerus ter [a] 9 metitur. Si autem poſt novenarium duos reliquero, qui mihi poſt illos incurrerit;

[a] Conjecturâ pro *tertio*.

rit, a primo metiendus eſt, per ſecundi imparis quantitatem; id eſt, per quinarium: nam ſi poſt 9 duos relinquam, id eſt 11 & 13, ternarius numerus 15 metietur, per ſecundi numeri quantitatem, id eſt, per quinarii; quoniam numerus ternarius 15 quinquies metitur. Rurſus, ſi a quindenario inchoans duos intermiſero, qui poſterior poſitus eſt, ejus primus numerus menſura eſt, per tertii imparis pluralitatem: nam ſi poſt 15 intermiſero 17 & 19, incurrit 21, quem ternarius numerus ſecundum ſeptenarium metitur; 21 enim numeri ternarius ſeptima pars eſt: atque hoc in infinitum faciens, reperio primum numerum, ſi binos intermiſero, omnes ſequentes poſt ſe metiri, ſecundum quantitatem poſitorum ordine imparium numerorum. Si vero quinarius numerus, qui in ſecundo loco eſt conſtitutus, velit[b] quis, cujus prima ac deinceps ſit menſura, invenire, tranſmiſſis quatuor imparibus, quintus ei quem metiri poſſit, occurrit. Intermittantur enim quatuor impares, id eſt, 7 & 9, & 11 & 13, poſt hos eſt quintus decimus quem quinarius metitur, ſecundum primi ſcilicet quantitatem, id eſt, ternarii; quinque enim 15 ter[c] metiuntur: ac deinceps, ſi quatuor intermittat, eum qui poſt illos locatus eſt, ſecundus, id eſt, quinarius, ſui quantitate metitur: nam poſt quindecim intermiſſis 17 & 19, & 21 & 23, poſt eos 25 reperio, quos quinarius ſcilicet numerus ſuâ pluralitate metitur; quinquies enim quinario multiplicato, 25 ſuccreſcunt; ſi vero poſt hunc quilibet quatuor intermittat, eâdem ordinis ſervatâ

[b] Conjecturâ pro *vel.*
[c] Conjecturâ pro *tertio.*

conſtantiâ, qui eos ſequitur, ſecundum tertii, id eſt, ſeptenarii numeri ſummam, a quinario metitur: atque hæc eſt infinita proceſſio. Si vero tertius numerus quem metiri poſſit exquiritur, ſex in medio relinquentur; & quem ſeptimum ordo monſtraverit, hic per primi numeri, id eſt, ternarii quantitatem metiendus eſt: et poſt illum, ſex aliis interpoſitis, quem poſt eos numeri ſeries dabit, per quinarium, id eſt, per ſecundum, tertii eum menſura percurret: ſi vero alios rurſus ſex in medio quis relinquat, ille, qui ſequitur, per ſeptenarium ab eodem ſeptenario metiendus eſt; id eſt, per tertii quantitatem; atque hic uſque in extremum ratus ordo progreditur. Suſcipient ergo metiendi viciſſitudinem, quemadmodum ſunt in ordine naturaliter impares conſtituti: metientur autem, ſi per pares numeros, a binario inchoantes, poſitos inter ſe impares, ratâ intermiſſione, tranſiliant; ut primus duos, ſecundus quatuor, tertius ſex, quartus octo, quintus decem [d]: vel ſi locos ſuos conduplicent, & ſecundum duplicationem terminos intermittant; ut ternarius, qui primus eſt numerus, & Unus, omnis enim primus Unus eſt, bis locum ſuum multiplicet, faciatque bis unum; qui cum duo ſint, primus duos medios tranſeat. Rurſus ſecundus, id eſt, quinarius, ſi locum ſuum multiplicet, 4 explicabitur: hic quoque quatuor [e] intermittat. Item ſi ſeptenarius, qui tertius eſt, locum ſuum duplicet, ſex creabit; bis enim 3 ſenarium jungunt: hic ergo in ordine [f] ſex relinquat. Quartus quoque, ſi locum

[d] Conjecturâ reſtitui pro 12.
[e] Conjecturâ pro 4.
[f] Conjecturâ pro *ordinem*.

 ſuum

ſuum duplicet, 8 ſuccreſcent; ille quoque octo tranſiliat: atque hoc quidem in cæteris perſpiciendum. Modum autem menſionis, ſecundum ordinem collocatorum, ipſa ſeries dabit. Nam primus primum quem numerat, ſecundum primum numerat [g], id eſt, ſecundum ſe; & ſecundum primus quem numerat, per ſecundum numerat [g], & tertium per tertium, & quartum item per quartum. Cum autem ſecundus menſionem [h] ſuſceperit, primum quem numerat ſecundum primum metitur; ſecundum vero quem numerat per ſe, id eſt, per ſecundum; & tertium per tertium: & in cæteris eâdem ſimilitudine menſura conſtabit. Illos [i] ergo ſi reſpicias, vel qui alios menſi ſunt, vel qui ipſi ab aliis metiuntur, invenies omnium ſimul communem mênſuram eſſe non poſſe, neque ut omnes quemquam alium ſimul numerent; quoſdam autem ex his ab alio poſſe metiri, ita ut ab uno tantum numerentur [k]; alios vero, ut etiam a pluribus; quoſdem autem, ut præter Unitatem eorum nulla menſura ſit. Qui ergo nullam menſuram præter Unitatem recipiunt, hos Primos & Incom-

[f] Conjecturâ pro 8.

[g] Pro *numerat* mallem in utroque loco, *metitur*, ut aliud ſit *numerare*, aliud *metiri*, & ſenſus ſit, "That which the firſt "number [of the Series] *counts* the firſt [of its multiples], it "*meaſures* by the firſt [of the Series], i. e. by itſelf. That "which it *counts* the ſecond [of its multiples], it *meaſures* by "the ſecond [number in the Series]." Sic enim infra legimus de Numero ordine ſecundo, "primum quem *numerat* ſeundum "primum *metitur*."

[h] Conjecturâ, pro *manſionem*.

[i] Conjectura, pro *alios*.

[k] Ang. "But ſo as to be counted in among the multiples of "one number only."

I   poſitos

positos judicamus; qui vero aliquam mensuram præter Unitatem, vel alienigenæ partis vocabulum sortiuntur, eos pronunciemus Secundos atque Compositos. Tertium vero illud genus, per se Secundi & Compositi, Primi vero & Imcompositi ad alterutrum comparati, hâc inquisitor ratione reperiet. Si enim quoslibet primos [l] numeros, secundum suam in semetipsos multiplices quantitatem, qui procreantur, ad alterutrum comparati, nullâ mensurâ communione junguntur: 3 [m] enim & 5, si multiplices, 3 ter [n] 9 faciunt, & quinquies 5 reddunt 25. His igitur nulla est cognatio communis mensuræ. Rursus 5 & 7 quos procreant, si compares, hi quoque incommensurabiles erunt: quinquies enim 5, ut dictum est, 25, septies 7 faciunt 49; quorum mensura nulla communis est, nisi forte omnium horum procreatrix & mater Unitas [o].

[l] Conjectura pro *illos*.

[m] Conjecturâ, pro *tres*. [n] Conjecturâ pro *tres tertio*.

[o] Sed cave credas, Lector, numeros inter se primos nullos dari præter Primorum Quadratos.